

Inventory Control Policies for Substitutable Deteriorating Items Under Quadratic Demand

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ABSTRACT

In today's busy life, no one has sufficient time to buy or bargain any type of products. It is common in such cases that customers, who are looking for purchasing a certain product, will be willing to substitute with a comparable product when facing a stock-out, rather than visiting a different store to bargain the original product. In this article, we study an inventory control problem in which demand is fulfilled by using two similar substitutable items. Retailer fulfilled demand of one product by other substitutable product when stock-out of the one of them. We consider inventory levels of both of the items and time dependent demand. The orders for both products are placed at the same time. Our objective is to maximize joint profit for two substitutable deteriorating items with respect to cycle time and a time at which one product is stock-out. The numerical analysis is carried out based on the analytical results. The critical inventory parameters are computed for the decision maker.

Keywords: *inventory control, constant deterioration, quadratic demand, substitutable products*

1. INTRODUCTION

In this throat-cut business world, there are many substitutable products in the market for competition. As we know that at any trades or superstore the occurrence of provisional stock-outs is a very common sensation in the classifications of often purchased items and it is also very common to see at any trades or superstore, consumers who eager to buy certain products will be willing to buy the substitute products, if they confronted the situation of the stock-outs. A survey report of Anupindi *et al.* (1998) also perceived the same sensation, in which he found that 82–88% of consumer would be willing to buy the substitute items if the desired items are out of stock. The substitutable items in which adequate deterioration can take place during the normal storage period of the units and subsequently this loss must be taken into account when analysing the inventory

system of substitutable items, i.e. the effect of deterioration plays a vital role in the choice of ordering amount of substitutable deteriorating items. When substitution will take place an additional cost is incurred, known as substitution cost. Such substitution costs may arise due to a diversity of causes: the cost of the reworking required on an item to make it substitutable for the other, loss of a customer's goodwill due to substitution, etc. The demand of smart phone remains in the market. When next generation smart phone is launched, its demand increasing linearly due to its features for certain time and then gets maturity in demand and later on demand starts decreasing exponentially due to new arrivals. The model proposed here can also be applied to automobiles industry, computer industry etc. In day to day life, one can visualize for the small scale retailers of milk products. Very few research is focussed on this concept. So, we consider in this paper quadratic demand. There are some citations in which they have taken quadratic demand.

In a general practice, demand rarely remains constant for infinite horizon. Shah *et al.* (2011) estimated a coordinated decision with two level credit limit for quadratic demand, which can be seen in seasonal items, fashion goods etc. It is observed in the market that with the presentation of contemporary generation product, market demand of the previous product decreases enormously. Shah *et al.* (2013) determine an optimal shipments, ordering, and imbursement policies for integrated vendor-buyer inventory system with net credit price-sensitive trapezoidal demand. Shah *et al.* (2015) evaluated an optimal down-stream credit period and cycle time for deteriorating inventory in a supply chain. Later, Shah *et al.* (2016) studied the model on the impact of future price increase on ordering policies for perishable items under quadratic demand. Recently, Shah *et al.* (2017) determined a deteriorate inventory model with expiration date of items under two level trade credit and preservation technology investment for time and price sensitive demand with DCF approach.

Deterioration of substances likes, explosive liquors, fruitless, taters, brews, remedy, blood etc. in form of conventional deterioration or damage, on-going corporeal decay in course of time, or desuetude is a natural existence and it has extensive effect on the player's inventory strategies. Ghare and Scharender (1963) and Raafat (1991) studied model for deteriorating item. Aggarwal and Jaggi (1995) developed inventory policies for deteriorating items under trade credit. Goyal and Giri (2001) studied inventory model for deteriorating item on recent drifts. Later, Yang (2005) estimated a model of comparison among numerous partial backlogging inventory models for deteriorating item. Pattnaik (2012) established a note on non-linear profit-maximization entropic order quantity (EnOQ) model for deteriorating items with stock dependent demand rate. Dye and Hsieh (2012) given a concept of preservation technology for deteriorating items. Avinadav *et al.* (2013) develop an inventory model for a perishable item with demand function sensitive to price and time. Dye (2013) estimated the effect of preservation technology investment on a non-instantaneous perishable inventory model. Taleizadeh and Nematollahi (2014) projected an inventory control problem for perishable items with financial deliberations and back-ordering. Fauza *et al.* (2015) analyzed a vendor-buyer inventory model for food products based on shelf-life pricing. Rabbani *et al.* (2015) instituted the model for supply chain of non-instantaneous stock dependent deterioration. Shah and Chaudhari (2015) studied an optimal policies time and credit dependent demand for three players with fixed life time and two-level trade credit. Furthermore, Zhang, *et al.* (2015) analyzed an integrated model for deteriorating item with revenue sharing and cooperative investment contract. Recently, Shah and Jani (2016) determined an inventory model for perishable item of fixed-life time with quadratic demand and up and down stream trade credit.

The firstly introduced inventory model of substitutable item by McGillivray and Silver (1978) by considering that all of the substitutable products have the same unit variable cost and shortage forfeit. Pasternack and Drezner (1991) formulated numerically that if the products are not substitutable then the related optimal order quantities can be enough. Furthermore, Drezner *et al.* (1995) established an EOQ model with comparison between two substitution and non-substitutable products. Gurnani and Drezner (2000) extended the model of Drezner *et al.* (1995) for multiple items. Mishra and Raghunathan (2004) provided new clarification for why buyers might be interested in vendor-managed inventory and showed that vendor-managed inventory exaggerates the competition between two makers of competing brands. In continuation of this, Schulz and Telha (2011) developed a model for the JRP with constant demands may have no polynomial-time algorithm. Krommyda *et al.* (2015), Salameh *et al.* (2014), Rasouli and Kamalabadi (2014), and Gerchak and Grosfeld (1999) advanced inventory model for two substitutable item with deterministic demand, fixed ordering cost and constant holding cost but no one deliberated the effect of deterioration in inventory decision of substitutable items. Zhao *et al.* (2014) determined the pricing policy for two substitutable products with price-dependent probabilistic demand with fixed ordering cost and constant holding cost. See **Table 1** for a comparison of existing models and our model based on the service level constraint.

Table 1: Contribution of different authors

	Deterioration	Time dependent demand	Substitutable items
Aggarwal and Jaggi (1995)	√		
Anupindi <i>et al.</i> (1998)			√
Avinadav <i>et al.</i> (2013)	√	√	
Drezner <i>et al.</i> (1995)			√
Dye and Hsieh (2012)	√		
Dye (2013)	√		
Gerchak and Grosfeld-Nir (1999)			√
Ghare and Scharender (1963)	√		
Goyal and Giri (2001)	√		
Gumani and Drezner (2000)			√
Pasternack and Drezner (1991)			√
Pattnaik (2012)	√		
Rabbani <i>et al.</i> (2015)	√		
Rasouli and NakhaiKamalabadi (2014)			√
Taleizadeh and Nematollahi (2014)	√		
Yang (2005)	√		
Zhao <i>et al.</i> (2014)			√
This Study	√	√	√

In this article, we consider stock and time dependent quadratic demand with constant deterioration of two substitutable items. In addition, we study an inventory control problem in which demand is fulfilled by using two similar substitutable items. Retailer fulfilled demand of one product by other substitutable product when stock-out of the one of them. We consider inventory levels of both of the items and time dependent demand. The orders for both products are placed at the same time. Under above assumptions, the objective is to maximize the total profit of the retailer with respect to cycle time and a time at which one product is stock-out.

The remaining part of the paper is organized as follows. Section 2 is the notations and assumptions used for the development of the proposed problem. We derived the model in Section 3. Section 4 represents the solution method to obtain the optimal policy. The numerical study and sensitivity analysis about key parameters is carried out in Section 5. Finally, Section 6 includes the conclusion of the study and its future scopes.

2. NOTATIONS AND ASSUMPTIONS

The proposed inventory problem is based on the following notations and assumptions.

2.1 Notations

- a_i Total market potential demand for the i^{th} product, $a > 0$, where $i = 1, 2$
- b_i Linear rate of change of demand for the i^{th} product, $b \geq 1$, where $i = 1, 2$
- c_i Quadratic rate of change of demand for the i^{th} product, $c_1 \geq 1$, where $i = 1, 2$

e_i	Rates of inventory level
x	The fraction of demand of the product that has stocked-out that will be substituted by the other product during stock-out, $0 < x \leq 1$
C_{1i}	Purchase cost of the i^{th} product per (\$/unit), where $i = 1, 2$
p_i	Procurement price for the i^{th} product (\$/unit), $p_i \geq C_{1i}$, where $i = 1, 2$
θ_i	Constant deterioration rate for the i^{th} product, where $i = 1, 2$
Q_i	Initial order quantities for the i^{th} product
q	Remaining quantities of product 2 after cycle time
$I_{ri}(t)$	The inventory level of the retailer at any time t for the i^{th} product
h_i	Holding cost rate (\$/unit / unit time) for the i^{th} product, where $i = 1, 2$
A_r	Fixed ordering cost per lot (\$/lot)
T	Cycle time (unit time) (decision variable)
$\pi_r(T, t_1)$	The total profit per unit time

Relations between parameters:

- $0 \leq \theta_1, \theta_2 < 1$
- $C_{1i} \leq p_i$

The problem is expressed as follows:

$$\text{Max } \pi_r(T, t_1)$$

2.2 Assumptions:

1. The inventory system involves substitutable deteriorate items.
2. The demand rate, for each product respectively, is a function of time and instantaneous stock-level $I_{ri}(t) > 0$ of both products at a certain time. The demand functions of the inventory system are given by $R_1(t, I_{r1}(t), I_{r2}(t)) = a_1(1 + b_1t - c_1t^2) + e_1I_1(t) - e_2I_2(t)$ and $R_2(t, I_{r1}(t), I_{r2}(t)) = a_2(1 + b_2t - c_2t^2) - e_1I_1(t) + e_2I_2(t)$ where, $a_1 = 0$ is total market potential demand, $0 \leq b_i \leq 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c_i < 1$ denotes the quadratic rate of change of demand and $e_i > 0$ is rates of stock level of each product. ($i = 1, 2$)
3. The units in inventory system of each player are subject to deterioration at a constant rate. The deteriorated units are not repaired or replaced during the cycle time.
4. At time t_1 the inventory level of one of the products reaches zero and during $t_1 \leq t \leq T$, a known fraction x of the excess demand is met by using the stock of the other product.
5. The ending inventory level of the second product could be greater than or equal to zero.
6. The planning horizon is infinite which will facilitate long time agreement.
7. Lead time is zero or negligible.

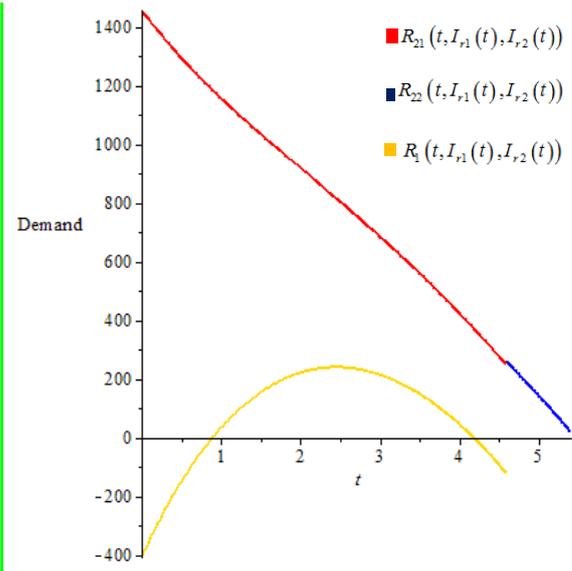


Figure 1 Demand of product 1 and 2 during cycle time

3. MATHEMATICAL MODEL

Consider an organization which sells two mutually substitutable products which deteriorate at a constant rate during a cycle time $[0, T]$. One product named as a product 1 and another as a product 2. At the starting of cycle time organization orders $Q_i (i = 1, 2)$ units of each product respectively. Then the stock level of the both product diminish due to demand. We assume that demand of the inventory system is function of time and corresponding stock level of the both the products at a certain time. If at time t_1 the inventory level of one of the products reaches zero, a known fraction of the demand for that product is fulfilled by using the stock of the other product. The replenishment cycle ends when the inventory level of the second product reaches to zero. In order to conduct the analysis of the problem, without loss of generality, we assume that product 1 is the first to stock-out. The inventory levels for product 1 and 2, respectively, at time t satisfy the following differential equations:

$$\frac{dI_{r1}(t)}{dt} = -R_1(t, I_{r1}(t), I_{r2}(t)) - \theta_1 I_{r1}(t) \quad 0 < t < t_1$$

with the condition $I_{r1}(t_1) = 0$

After depleting the product 1 organization sells product 2 to fulfilled demand of product 1.

$$\frac{dI_{r2}(t)}{dt} = -R_2(t, I_{r1}(t), I_{r2}(t)) \quad 0 < t < t_1$$

During the time interval $[t_1, T]$, the inventory level of product 1 is vanished and product 2 can be represented by the following differential equation

$$\frac{dI_{r2}(t)}{dt} = -R_2(t, I_{r1}(t), I_{r2}(t)) - x \cdot R_1(t, I_{r1}(t), I_{r2}(t)) \quad t_1 < t < T$$

With boundary condition $I_{r2}(T) = q$ and $I_{r2}(t_1^-) = I_{r2}(t_1^+)$

Solving above differential equations, we get inventory level at any time t

$$I_{r1}(t) = C_2 e^{\beta_2 t} - e^{\beta_2(t-t_1)} (C_2 e^{\beta_2 t_1} + A_6 t_1^2 + A_7 t_1 + A_8) + A_6 t^2 + A_7 t + A_8, \quad 0 \leq t \leq t_1$$

$$I_{r2}(t) = \frac{1}{e_2} \left(C_2 \beta_2 e^{\beta_2 t} - \beta_1 e^{\beta_1(t-t_1)} (C_2 e^{\beta_2 t_1} + A_6 t_1^2 + A_7 t_1 + A_8) + 2A_6 t + A_7 + a_1(-c_1 t^2 + b_1 t + 1) + (e_1 + \theta_1) (C_2 e^{\beta_2 t} - e^{\beta_1(t-t_1)} (C_2 e^{\beta_2 t_1} + A_6 t_1^2 + A_7 t_1 + A_8) + A_6 t^2 + A_7 t + A_8) \right), \quad 0 \leq t \leq t_1$$

$$I_{r2}(t) = \left(\frac{A_{10} + A_{11}t + A_{12}t^2}{A_9} \right) - \left(\frac{A_{11} + 2A_{12}t}{A_9^2} \right) + \frac{2A_{12}}{A_9^3} + e^{A_9(T-t)} \left(q - \left(\frac{A_{10} + A_{11}T + A_{12}T^2}{A_9} \right) + \left(\frac{A_{11} + 2A_{12}T}{A_9^2} \right) - \frac{2A_{12}}{A_9^3} \right), \quad t_1 \leq t \leq T$$

where,
 C_2

$$= \frac{1}{A_9^3(\beta_1 - \beta_2)} \left(\left(\left(\begin{matrix} -A_9^3 a_1 c_1 - A_{12} A_9^2 e_2 \\ -A_6 A_9^3 \beta_1 \end{matrix} \right) t_1^2 + \left(\begin{matrix} -2A_{12} e_2 - A_8 A_9^3 \beta_1 \\ + A_7 A_9^3 + A_9^3 a_1 \end{matrix} \right) \right) + \left(\begin{matrix} A_9^3 a_1 b_1 - A_{11} A_9^2 e_2 \\ + 2A_{12} A_9 e_2 + 2A_6 A_9^3 + A_7 A_9^3 \beta_1 \end{matrix} \right) t_1 + \left(\begin{matrix} -A_9^3 e_2 q + A_{10} A_9^2 e_2 - A_{11} A_9 e_2 \\ + 2A_{12} e_2 + A_{12} A_9^2 T^2 e_2 + \\ A_{11} A_9^3 T e_2 - 2A_{12} A_9 T e_2 \end{matrix} \right) e^{A_9(T-t_1)} \right) e^{-\beta_2 t}$$

For convenience, the following notations are introduced:

$$A_1 = (e_2 + \theta_2)(e_1 + \theta_1) - e_1 e_2, \quad A_2 = (e_1 + \theta_1) + (e_2 + \theta_2)$$

$$A_3 = -a_1 b_1 - a_2 e_2 - (e_2 + \theta_2) a_1, \quad A_4 = 2a_1 c_1 - a_2 e_2 b_2 - (e_2 + \theta_2) b_1 a_1$$

$$A_5 = a_2 c_2 e_2 + a_1 c_1 (e_2 + \theta_2), \quad A_6 = \frac{A_5}{A_1}$$

$$A_7 = \frac{A_1 A_4 - 2A_5 A_2}{A_1^2}, \quad A_8 = \frac{A_1^2 A_3 - 2A_1 A_5 - A_1 A_4 A_2 + 2A_5 A_2^2}{A_1^3}$$

$$A_9 = e_2 + \theta_2 + x e_2, \quad A_{10} = -(a_2 + x a_1)$$

$$A_{11} = -(a_2 b_2 + x a_1 b_1), \quad A_{12} = a_2 c_2 + x a_1 c_1$$

$$\beta_1 = \frac{-A_2 + \sqrt{A_2^2 - 4A_1}}{2}, \quad \beta_2 = \frac{-A_2 - \sqrt{A_2^2 - 4A_1}}{2}$$

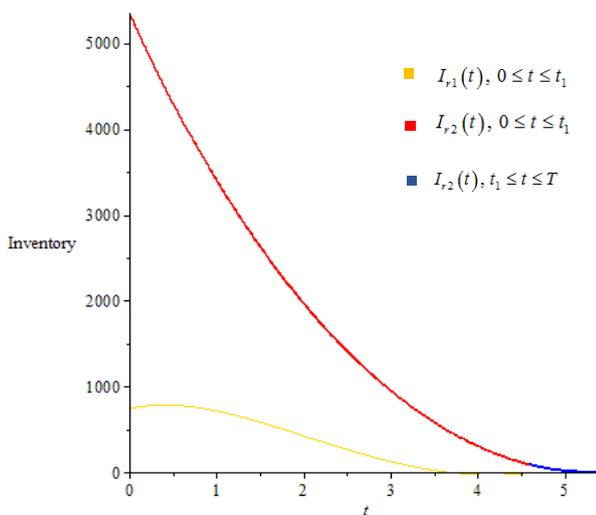


Figure 2 Inventory level of product 1 and 2 during cycle time

In Figure 1 and Figure 2, we have analyzed that after t_1 when product 1 is stocked out, demand of product 1 fulfilled by product 2.

Then, the relevant inventory costs can be calculated by considering the following components

- Ordering of inventory cost per unit time : $OC_r = \frac{A_r}{T}$
- Purchase cost of item per unit time : $PC_r = \frac{1}{T} \sum_{i=1}^2 C_{1i} Q_i$
- Holding cost per unit time : $HC_r = \frac{1}{T} \left[h_1 \int_0^{t_1} I_{r1}(t) dt + h_2 \left(\int_0^{t_1} I_{r2}(t) dt + \int_{t_1}^T I_{r2}(t) dt \right) \right]$

End of the cycle time retailer can earn sales revenue:

$$SR_r = \frac{1}{T} \left[\int_0^{t_1} (p_1 Q_1 + p_2 Q_2) dt + \int_{t_1}^T p_2 Q_2 dt \right]$$

where, $Q_1 = I_{r1}(0)$ and $Q_2 = I_{r2}(0)q$.

Therefore, the retailer's average profit for item is

$$\pi_r(T, t_1) = SR_r - OC_r - PC_r - HC_r$$

The total profit function $\pi_r(T, t_1)$ is a continuous function of cycle time T and a time at which product 1 is stock out. We will established endorsement of the proposed model using numerical example. The maximization of the total profit will be shown graphically for the obtained results.

4. NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

4.1 Numerical Example

Example: Consider the following parameters: scale demand of product 1 is $a_1 = 500$ units, linear and quadratic rates of demand for the product 1 are $b_1 = 20\%$, $c_1 = 10\%$, rates of inventory level of product 1 is $e_1 = 15\%$, the fraction of demand of the product that has stocked-out that will be substituted by the other product during stock-out is $x = 10\%$, holding cost rate of the product 1 is $h = \$2/\text{unit}/\text{unit time}$, combine ordering cost for product 1 and 2 is $A = \$50/\text{lot}$, purchase cost of the product 1 is $C_{11} = \$2/\text{unit}$, constant deterioration rate of the product 1 is $\theta_1 = 25\%$, and selling price of the product 1 is $p_1 = \$7$, whereas scale demand product 2 is $a_2 = 500$ units, linear and quadratic rates of demand for the product 2 are $b_2 = 25\%$, $c_2 = 10\%$, rates of inventory level of product 2 is $e_2 = 20\%$, holding cost rate of the product 2 is $h = \$1.5/\text{unit}/\text{unit time}$, purchase cost of the product 2 is $C_{12} = \$1.5/\text{unit}$, constant deterioration rate of the product 2 is $\theta_2 = 15\%$ and selling price of the product 2 is $p_2 = \$8$. Here we consider $q = 0$. Using Maple 18 we obtain the optimal values of the decision variables are cycle time $T^* = 5.304$ unit time and a time at which product 1 is stock out $t_1^* = 4.589$ unit time. So, finally total profit of the inventory system is $\pi_r^* = \$42335.98$. As shown in Figure 3, the unit total profit of the inventory system is concave.

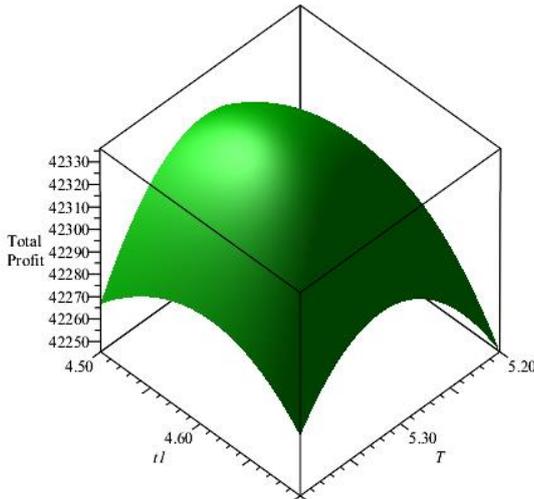


Figure 3 Concavity of the total profit with respect to T and t_1

Also, the concavity of the profit function is acquired by the well-known Hessian matrix. Now, Hessian matrix is for the above inventory system is

$$H(T, t_1) = \begin{pmatrix} \frac{\partial^2 \pi_r(T, t_1)}{\partial T^2} & \frac{\partial^2 \pi_r(T, t_1)}{\partial T \partial t_1} \\ \frac{\partial^2 \pi_r(T, t_1)}{\partial T \partial t_1} & \frac{\partial^2 \pi_r(T, t_1)}{\partial t_1^2} \end{pmatrix}$$

Using the above example, we get the hessian matrix $H(T, t_1)$ at the point (T^*, t_1^*)

$$H(T^*, t_1^*) = \begin{pmatrix} -8665.72 & 915.03 \\ 915.03 & -5199.87 \end{pmatrix}$$

As in Barrón and Sana (2015), if the Eigen values of the hessian matrix at the solution (T^*, t_1^*) are all negative, then the profit function $\pi_r(T^*, t_1^*)$ is maximum at that solution. Here, Eigen values of above Hessian matrix are $\lambda_1 = -8892.47$ and $\lambda_2 = -4973.13$. So, the profit function $\pi_r(T, t_1)$ is maximum.

4.2 Sensitivity Analysis

In **Table 2**, the sensitivity analysis of some critical inventory parameter of above example is carried out by varying one variable at a time as -20%, -10%, 10%, and 20%.

1. From the **Table 2**, it can be observed that holding cost rate for the 2nd product, purchase cost of both the products and constant deterioration rate for the 2nd product slowly increase the cycle time.
2. From the **Table 2**, it can be seen that holding cost rate for the 1st product, purchase cost of 1st product and constant deterioration rate for the 2nd product slowly increase whereas holding cost rate for the 2nd product and purchase cost of 2nd product slightly decrease t_1 .
3. From the **Table 2**, it can be analyzed that constant deterioration rate for the 1st product rapidly increase whereas holding cost rate and purchase cost for the 2nd product rapidly decrease total profit. Moreover, holding cost rate and purchase cost for the 1st product slowly decrease total profit.

Table 2 Sensitivity analysis

Inventory Parameter	Decision variables	Change (%)				
		-20%	-10%	0%	10%	20%
h_1	t_1 (unit time)	4.562	4.575	4.589	4.604	4.745
	π (in \$)	42462.32	42398.67	42335.98	42274.33	40134.45
h_2	T (unit time)	5.293	5.299	5.304	5.310	5.315
	t_1 (unit time)	4.596	4.592	4.589	4.586	4.582
C_{11}	π (in \$)	42853.26	42594.45	42335.98	42077.87	41820.12
	T (unit time)	5.301	5.303	5.304	5.305	5.307
C_{12}	t_1 (unit time)	4.574	4.581	4.589	4.597	4.605
	π (in \$)	42392.96	42364.32	42335.98	42307.95	42280.24
θ_1	T (unit time)	5.298	5.301	5.304	5.307	5.310
	t_1 (unit time)	4.594	4.591	4.589	4.586	4.584
θ_2	π (in \$)	42638.97	42487.41	42335.98	42184.68	42033.52
	π (in \$)	41241.96	41366.11	42335.98	42627.22	42953.53
θ_2	T (unit time)	5.272	5.290	5.304	5.315	3.964
	t_1 (unit time)	4.585	4.587	4.589	4.591	4.760

5. CONCLUSION

In this article, we studied inventory control problem of substitutable constant deteriorating items in which demand is fulfilled by using two similar substitutable items. Retailer fulfilled demand of one product by another substitutable product when stock-out of the one of them. We consider

inventory levels of both of the items and time dependent demand. The orders for both products are placed at the same time. We analysed with the help of substitutable item one can earn more profit as well as ignore shortages. For numerical examples, inventory control system reaches the maximum profit and carry-out sensitivity analysis with respect to inventory parameters. Current research have numerous

possible extensions like, model can be further generalized by taken more items at a time and more substitutable items also. One can also analysed multi layered supply chain. Research can be extended for finite or infinite planning horizons.

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